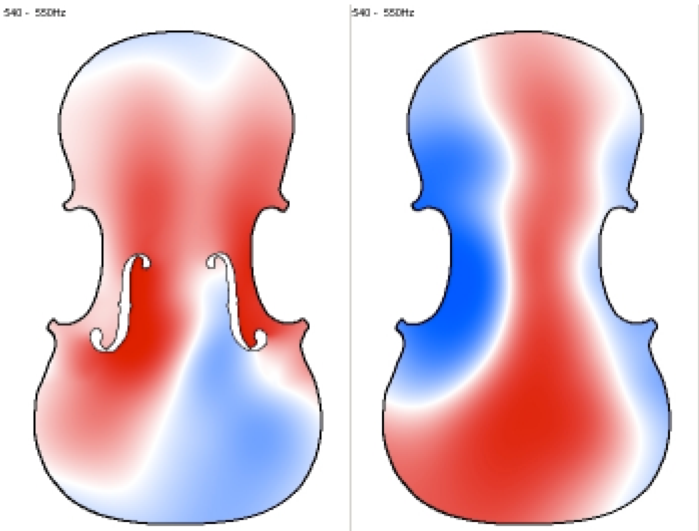
BENDING MAPS

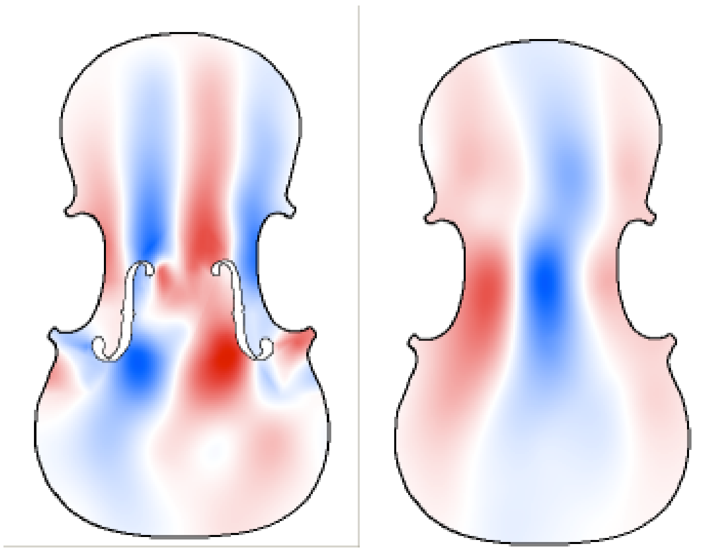
George Stoppani March 2009

Creating images indicating areas of high bending uses the same data collected for normal modal animations, and uses an interpolation algorithm to plot the relative deflections. We won't discuss here the details of how this is done but will concentrate on what they can show us. All plots show both plates as viewed from the outside.



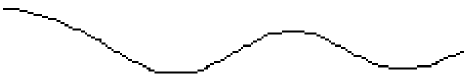
This is a band-averaged view of a B1+ mode (see, Band Averaged Movies, Strad3D)

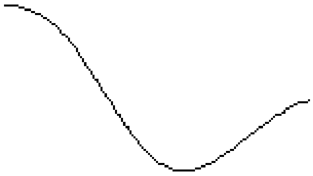
This is a well separated mode (low modal overlap) and the ODS averaged over 540 to 550 Hz (mode frequency is 548 Hz) is all but identical to the fitted mode shape.



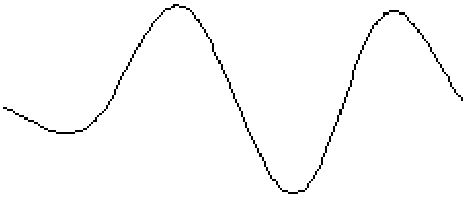
Here is the same B1+ mode shape but showing the cross-grain bending. The two colours indicate whether the bending is concave or convex and the intensity of colour the radius of curvature. A small radius is tight bending and will have strong colour whereas a flat surface has infinite radius and would be white.

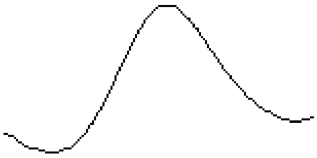
Below slices have been take through the upper bout of the top and through the waist of the back. (Viewed from the tail end, outside of top plate uppermost) These are the displacements and can be related to the bending plots.



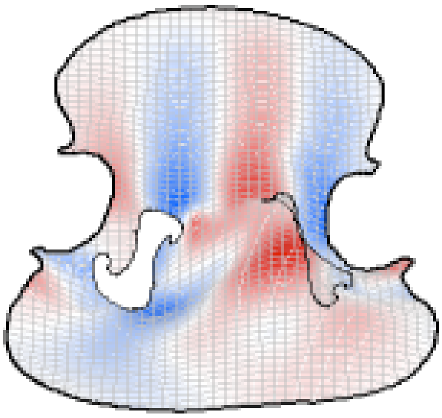


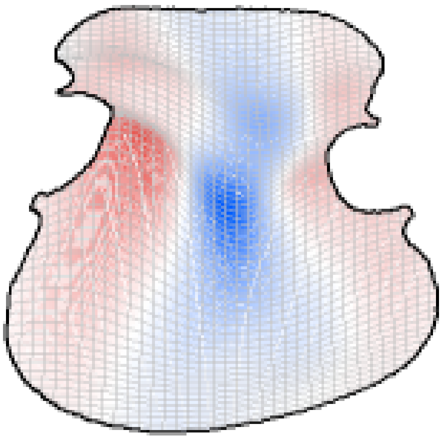
Below: the same slices showing the bending.





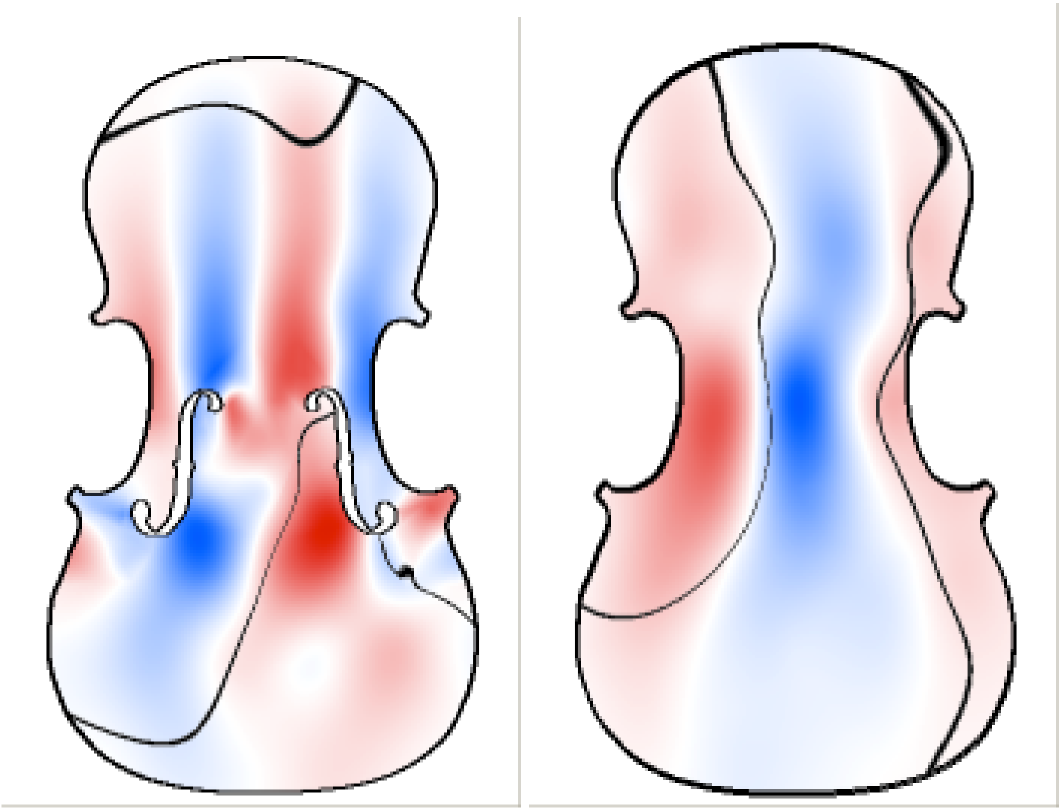
We can go a step further and superimpose the bending (shown in colour) into the mesh (showing displacement).





(See additional mode animations with bending in **Shapes of the Signature Modes**, Strad3D)

It is a common misconception that nodal lines are areas of no activity. They are areas of no spatial displacement with opposite phase either side, but they can and often do bend at these locations. Below I have pasted the nodal lines over the bending plots.

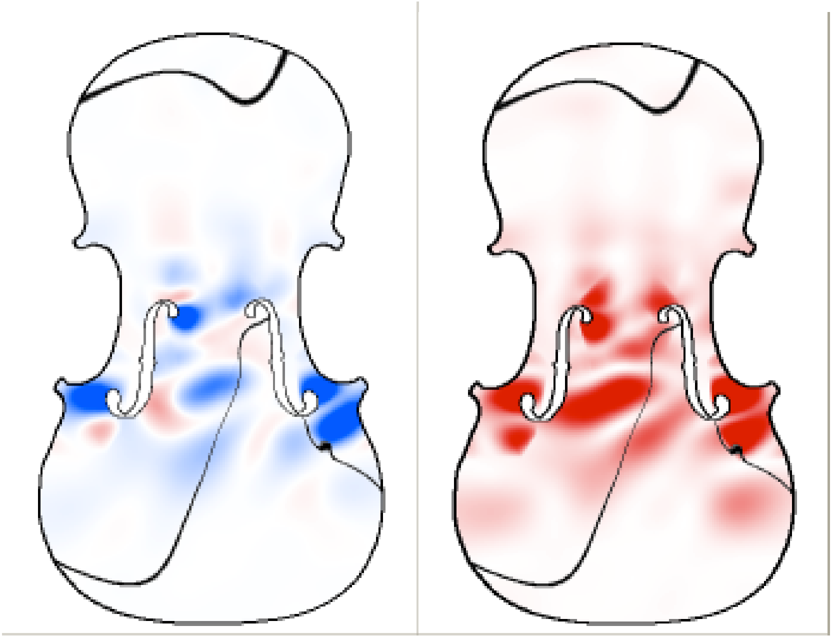


While there is a strong correspondence between the displacement and the bending for the back this is not so for the front. We would see more extreme examples in other modes. Modes store their energy alternately in the motion of the mass and in the spring energy of bending areas. If we could do the same for the in-plane motion we would see that energy is also stored in lateral distortion and motion. The Polytec 3D laser plots clearly show that in-plane motion is frequently significant.

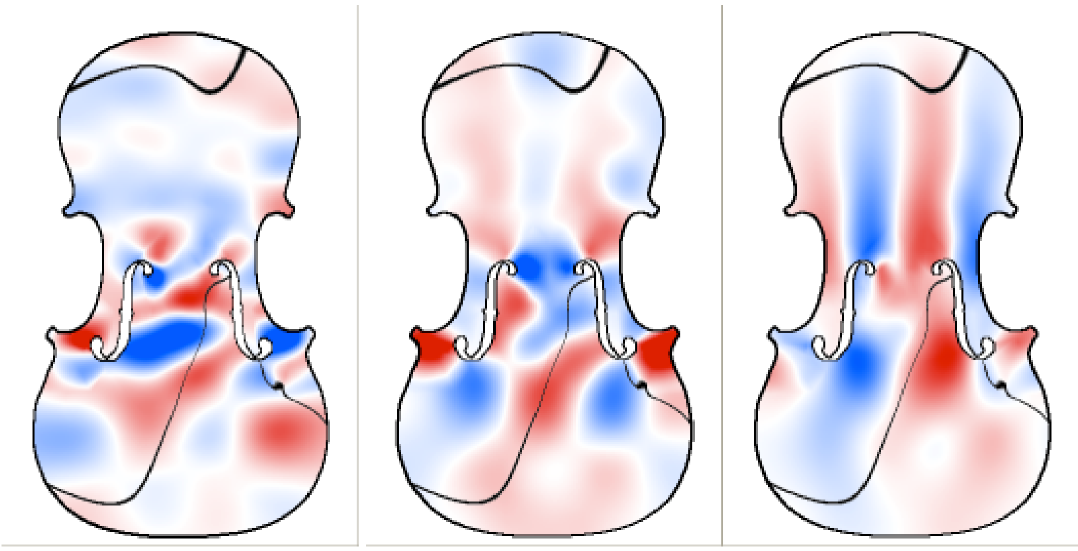
The bending is of interest in itself but also helps us to read the mesh plots. It is possible to show the long-grain bending, twist and combinations of these.

A theoretical model; Stored bending energy

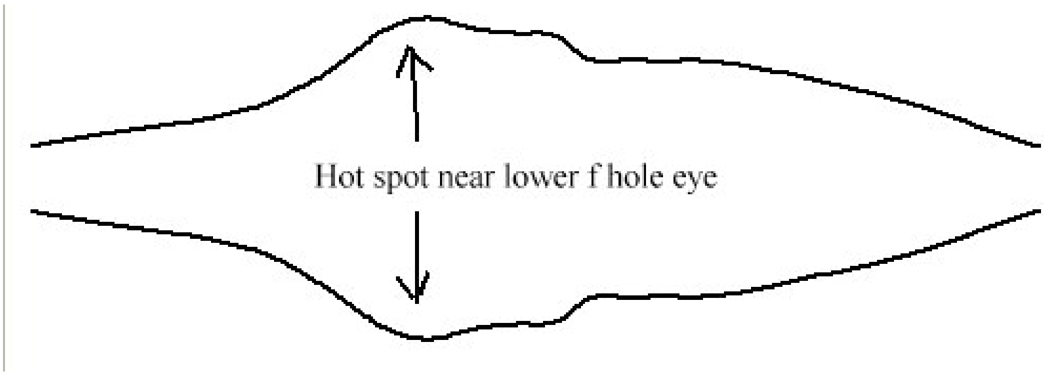
Trying to take this inquiry further, a recent speculative experiment has been to plot an estimate of the bending energy in the top plate. Because of the complexity of the task we have simplified the model by assuming it to be a flat plate made of quarter-cut spruce and with even thickness. This is, of course, not true for a real violin and we don't really know just how incorrect it is. It takes no account of the ribs, bass bar or arching. Clearly there will be more energy stored in stiffer areas such as along the middle of the bass bar. A combination of all the bending which represents the total out-of plane distortion of the top plate is shown (below left). Blue represents a ‘saddle’ – where the bending in one direction and at right angles to it have opposite signs. Where it is red the surface is either convex or concave. Below right is the same but weighted according to the stiffness parameters of spruce. This has positive values only (so no sign) since the stored energy is the same which ever way the bending goes.



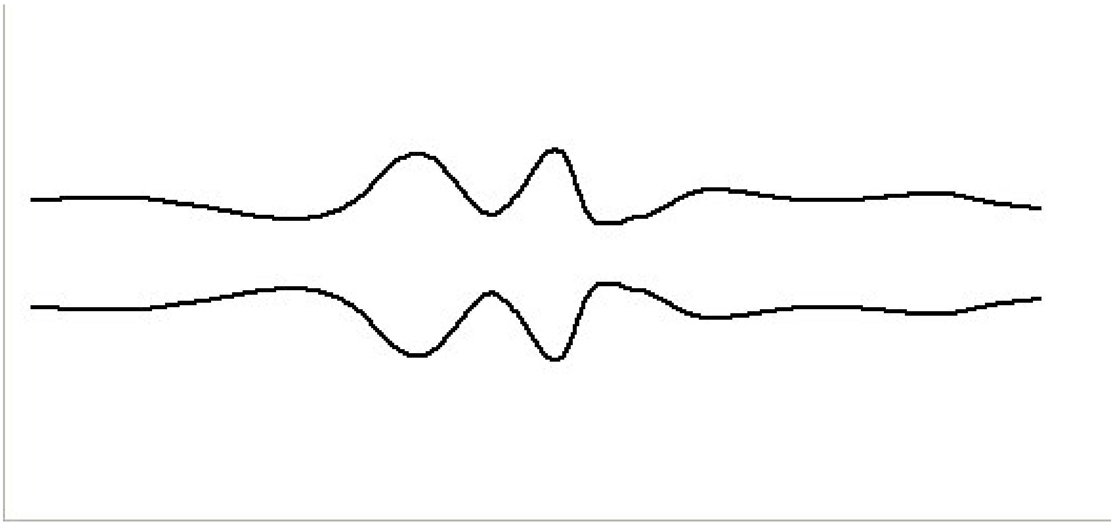
These results may seem surprising: the areas of high bending are discrete and sharply defined. The considerable difference for the flat-plate energy estimate can be explained by the large difference in long-grain stiffness and cross-grain stiffness. The bending tends to be dominated by the greater cross-grain flexibility but this stores relatively little energy compared to the long-grain bending. If we look at the long-grain bending (below left), the twist (below centre) and cross-grain bending (below right) we may be able to see what the bending energy represents. They are arranged in order of stiffness.



The complication in the long-grain bending is perhaps surprising. Below are both phases of a displacement slice taken approximately along the bass bar. (Tail block to the left and neck to the right)



And the long-grain bending:



Certainly part of the explanation is the weakening of the plate by the f holes particularly at the eyes. This allows the island to twist more than the areas above and below the holes. Another factor may the compression along the grain due to the string tension. A modal analysis of a violin with the strings tuned down (just enough tension to keep the bridge in place) appears to have less complicated long-grain bending than the same violin tuned to pitch.

Here we have looked at examples of only the B1+ mode but there are many other modes common to most violins each of which has a unique pattern of displacement and bending. Each instance of these modes in any violins is different, slightly or significantly. ~~~

Modal images created by George Stoppani with his suite of modal software;

Acquisition, ModeFit and **ModeShape**