This article is a general introduction to violin modes and acoustics from a violinmaker's viewpoint, and as such is not intended to be definitive or to fully cite all sources. The focus is on the low frequency modes, and many other important topics are omitted or skimmed over.

More About Modes

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1. Basic Introduction to Modes

Just as a microscope allows us to see things that are too small for the human eye, modal analysis is a technique for helping us to see how objects vibrate. We already know that violins make sound because they vibrate, but the movement is too small and too fast for us to see. If we hook up some sensors to the violin and a computer, then crunch it all through some elaborate software, we can have a visual representation of the vibration slowed down in time, and with very exaggerated movement. With some patience we can begin to grasp how violins make sound in more detail.

The first time we see modal animations, we will probably be surprised that the vibration patterns are not at all as we might have expected. This was certainly my own experience when I first saw rather primitive wire-frame animation at the second Tiverton conference back in 1987. I realised immediately that my own 'intuitive engineering' view was entirely wrong. I was bewildered by the complexity, and still am in many ways. However, I hold the view that leaving behind a wrong idea is almost as useful as acquiring new understanding that is actually correct.

An appropriate definition of what a mode is depends on what sort of system we are discussing. *{See note (1) below}* The example of a weight on a spring is often used to illustrate a system with a single mode, but a violin has a great many – at least 40 or 50 below 3kHz, though it is problematic to identify them all with confidence.

Fortunately for our understanding, all violins are in some respects the same. A large number of modes are common to all violins and have very similar shapes but at slightly different frequencies. In this sense all violins or even all boxes with holes in are the same. In another sense all violins are unique: no two have exactly the same modal properties because even if they are dimensionally very close no two pieces of wood are identical. A useful analogy might be the human face: all have eyes, nose and mouth and ears. Some are remarkably similar at a glance; others are very distinctly different, yet all have the same basic features and each is unique. The first step in overcoming our bewilderment is to learn to spot the eyes and noses in the vibration patterns, or shapes, that happen in particular frequency ranges in violins. We can worry about the moles, warts, stubble on the chin etc later.

What we need to realise first is that violins are very unlike high quality loudspeakers. They radiate very much more at some frequencies than others, and that at these frequencies where the radiation is strong there is always a vibrating shape that is the underlying cause.

2. Why should violinmakers be interested in modes?

A modal view, based on the vibrations of the wood and the air enclosed in the box, is a simpler model to understand than one based directly on the properties of the wood or the design of any particular violin.

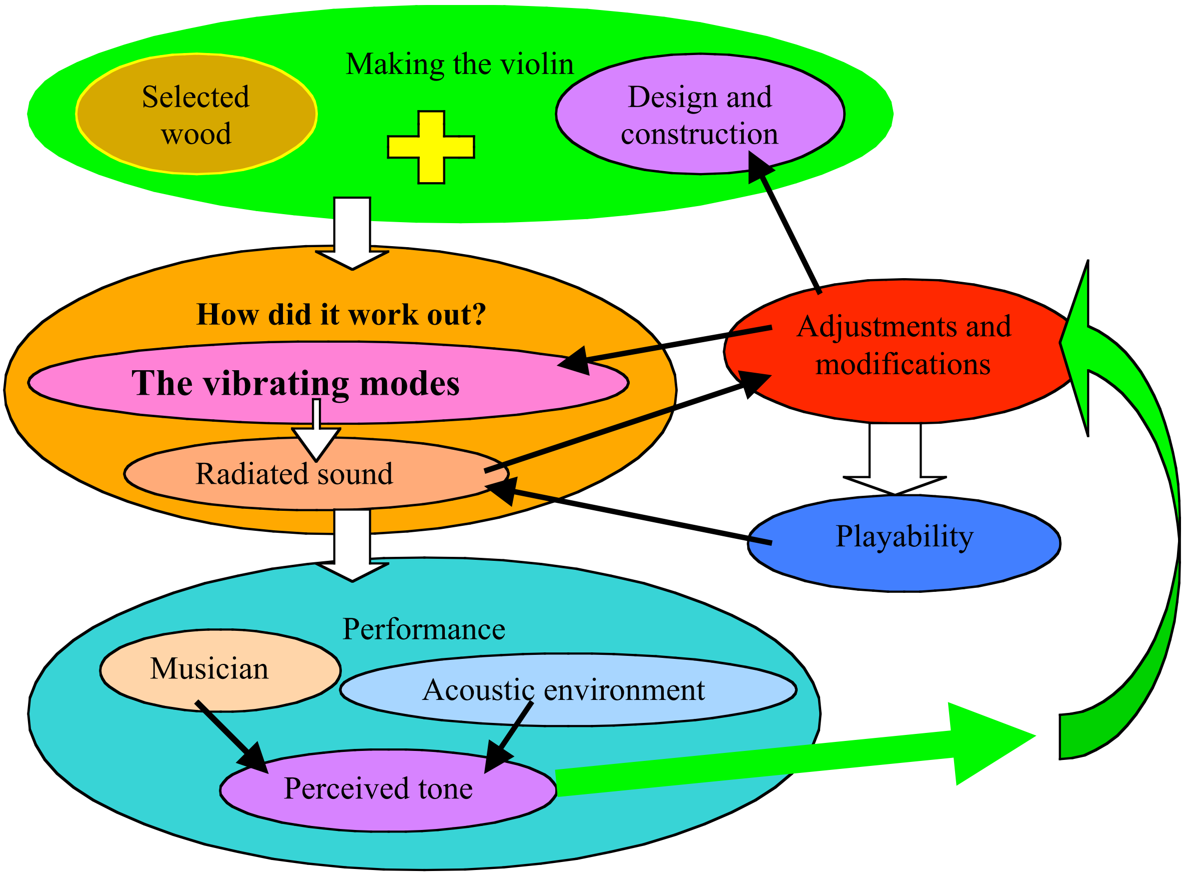
- The perceived tone of the violin is a consequence of the radiated sound.
- Violins radiate sound because the wood moves when the strings vibrate and cause the air to move.
- The level of vibration in the wood is a direct consequence of the modes that exist in that violin.
- The level of sound radiated at different frequencies relates to the level of vibration in the wood at those frequencies and how effectively the vibrating pattern radiates.
- These modes are themselves a consequence of the chosen shape, construction details and the mass, stiffness and damping properties of the wood. (These create the foundation on which simulations of violin vibration rest.)

Each of the above five statements requires some expansion, and we could play with the order or the wording. These are not new or controversial ideas, but serve to highlight the fact that a chain of consequences exists from the forest to the concert hall. More explicitly, the modal activity of the violin is the interface between the forest and the philharmonic ear. To one side lie axes or chain-saws, gouges and glue pots, and to the other a musician's skill and concert hall acoustics or the recording studio. This is the last detail that the maker can hope to control and the immediate cause of what is heard in performance. What is new is the relative ease with which we now can extract accurate and detailed information from assembled violins. We can now see clearly what was previously either invisible, seen only dimly or subject to erroneous beliefs.

With only an understanding of how mode shapes represent the motion of the wood at particular frequencies, a violin maker can get a feel for what actually happens – which parts move, how much they move relative to each other, and where the bending of the plates and whole corpus takes place. How this will affect their making practices is another matter, but at least they will be starting from fact rather than imagination.

Below is a bubble diagram to illustrate the causal connections and feedback loops. Modifications might be made during the making process, after the violin is strung up and at anytime thereafter. One cannot adjust the radiated sound directly but only via modifying the modes. The perceived tone is a result of both the radiated sound and the playability factors. *{See (3) in appendix below}* There will usually be a performance phase in the workshop with the maker as the musician. One might add another back-arrow that leads from the performance or final evaluation of the violin back into the

“Making the violin” bubble, i.e. the maker will revise his or her choice of wood and design details in the light of experience.



During the making stage the choice of wood, the selected model and other details of construction (particularly the plate arching) largely determine the outcome and only some modifications are possible. The “How did it work out?” bubble is the violin ready to be played. It will change with subsequent modifications and over time. The musician, at any point during or after the purchasing process, may go back to the maker and ask for adjustments. The question is “just how good can a maker be at modifications and adjustments both during and after the making process?” The argument presented is that knowledge of violin modes in general and, even better, information about the modes of a particular violin that needs modification, can lead to much more effective intervention. Many makers working at this time measure and record wood properties, plate weights, tap tone frequencies and graduations. Some also measure the sound radiation and monitor the frequencies of major modes. A few actually do modal analysis as a part of their making process.

The perceived tone of the violin is a consequence of the radiated sound

Let’s start with the most problematic part of the chain and make a distinction between acoustic radiation measured in a neutral way and what is heard when the violin is played. *{See note (2) below}* Given suitable equipment and procedures we can make a variety of measurements of violin acoustic radiation that are meaningful in a scientific sense. However, "perceived tone" (what the player or listener hears) is not directly measurable at all. It can only be approached indirectly by means of controlled listening tests and statistical exercises. Such experiments demonstrate firstly that most people, including musicians and violin-makers, are neither very accurate nor consistent in their judgements. No two people share exactly the same perceptions or have the same understanding of the words commonly used to describe tone.

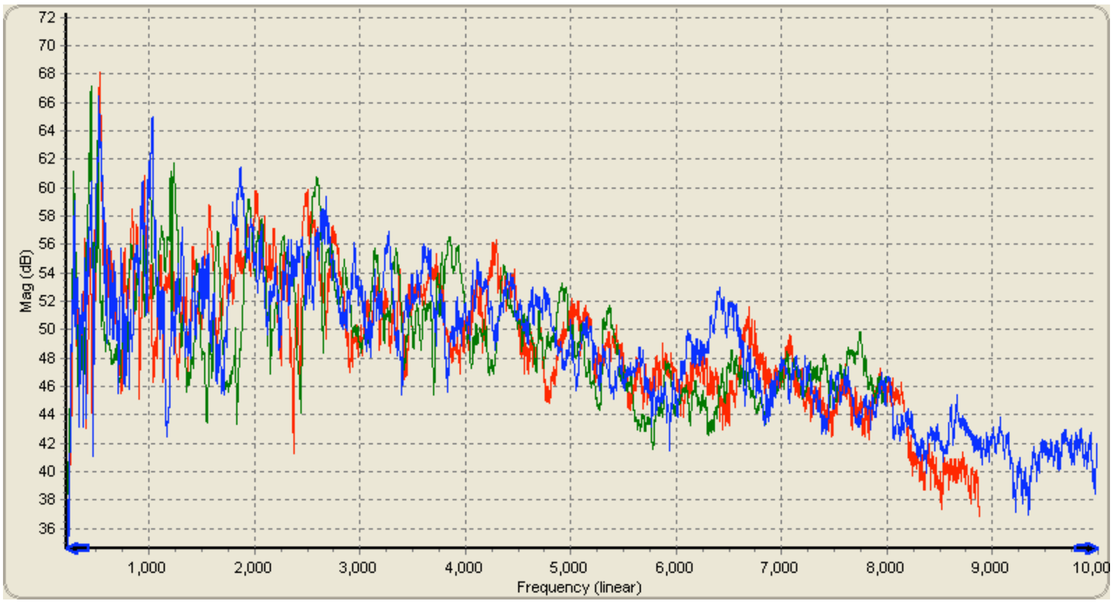
While this makes the task of quantifying violin tone extremely difficult, it is not the case that judgements are impossible or meaningless. In a sound adjustment session a musician and maker will often notice very subtle differences. In this situation it may well be "playability" issues that dominate and not small changes in frequency content or response levels to input force. Again these are rather subjective matters and we do not expect exactly the same judgements from all musicians. It seems likely that most musicians do not and don't feel any need to put tone quality and playability in separate compartments.

Psychoacoustics (the psychology of perception applied to hearing) is now a major area of violin research. Recent work *{see note (4)}* has revealed information about what sort of differences in frequency content we can and cannot hear. A fundamental problem was that we were looking at the spectra without knowing which features we could even hear or which features we are most sensitive to. Trying to interpret a radiation spectrum can be exasperating: two violins may sound very different yet have remarkably similar spectra or their spectra may appear very different while the sound is very similar. We might adjust a soundpost and perceive a change in sound and feel to the player while the spectra from before and after don't seem to back up what we perceive, or might even suggest the opposite. In other words we don't really know that much about what to look for in a spectrum beyond rather basic features.

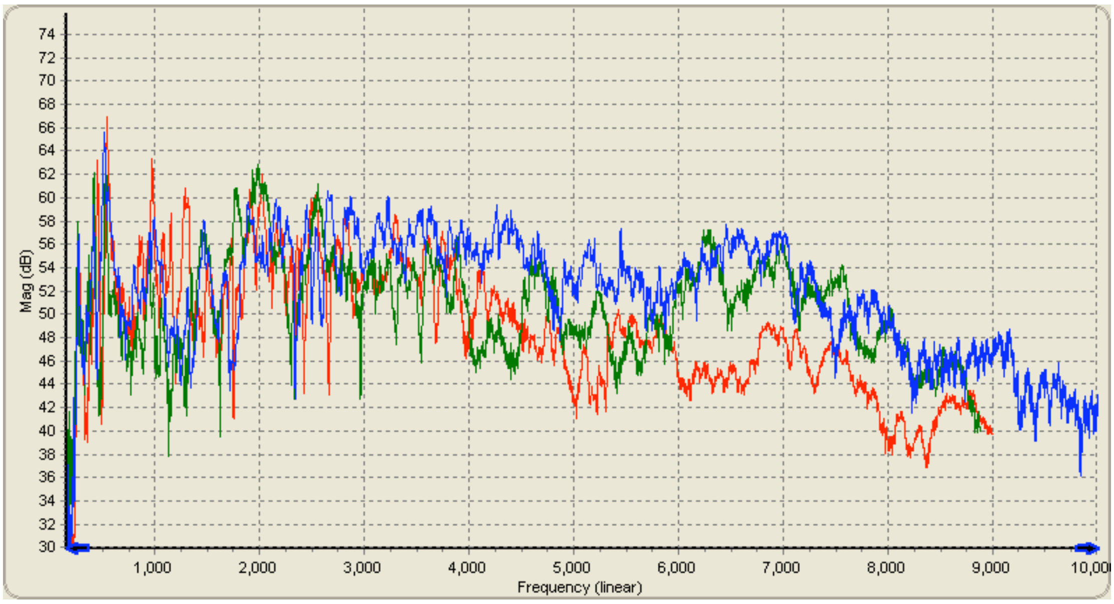
Nobody would suggest that violin tone can or should be assessed solely on the basis acoustic radiation measurements. If we ask a great player with a fine Strad why they like that violin, the answer will probably not be "just the raw power". Of course, the ability to be clearly heard is vital, but this quality can be found in violins that cost a fraction of the price. The answer is likely to contain something like "it's what I can do with it".

Like tone quality, playability is not directly measurable and it is not studied as much as other areas of violin research. Playability must nevertheless be a consequence of the violin's modes, though strings, bow and adjustments can make a large difference. While playability issues are clearly very important, they are only a part of what makes a good violin. . If something is missing in the violin's frequency response spectrum, good playability can only compensate up to a point. However, when the acoustic radiation is measured by a method that leaves out the musician we have effectively separated tone and playability. What we want to know is what features are desirable or undesirable in this spectrum, and we do know something about this.

Below are two charts showing overlays of the acoustic radiation of 3 del Gesus and 3 Strads. All are excellent violins: they are all somewhat different from each other and there is also a difference between del Gesus and Strads. One could almost draw a straight line sloping downwards through del Gesus whereas the Strads tend to be more level. (We could find exceptions that do not fit this pattern.) Clearly there is no simple or single formula for what constitutes a good violin though it is clear that overall the levels need to fairly high and that there must be strong radiation both in the low frequency end and in the high end.



Acoustic radiation of three del Gesu violins.



Acoustic radiation of 3 Strads.

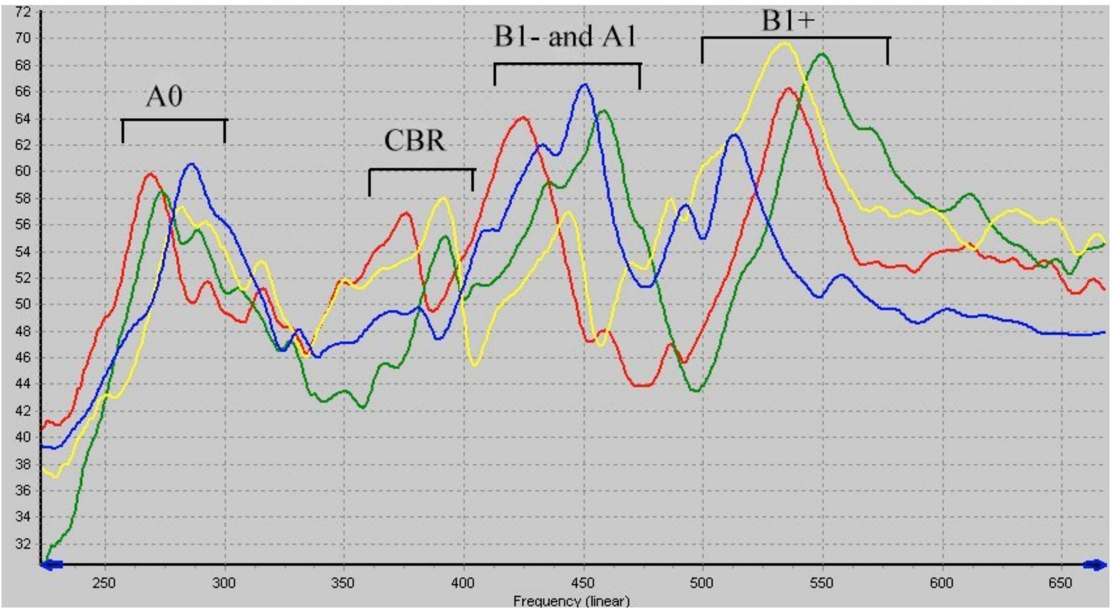
The human ear is much more sensitive to some frequency regions than others and to the relative levels in different regions. In discussion of the violin resonance profile, the work of Heinrich Dünnwald is often quoted. While looking at the details of a spectrum can be confusing, Dünnwald looked at spectra in terms of averages within frequency bands, which makes the differences clearer.

In addition to collecting data generated by an impact hammer, other approaches include analysing the recorded sound of a violin being played, and averaging the frequency content over an extended period of time (Long Time Average, LTA). Though dependent on room acoustics, recording technique and the musical excerpt, it provides another useful view, and can be used to analyse readily available existing recordings. Transients (or the beginnings of bowed notes) are another important but hard to measure aspect of sound, and ongoing experiments seek to examine the changes in frequency content over these very small time intervals, through digital signal processing. It has also been suggested that directionality is important. Directionality could mean in a simple sense how much radiates from the front compared to the back and differences have been observed between violins. At low frequencies violin radiation is predominantly "monopole": it spreads like an expanding sphere equally in all directions. At higher frequencies it becomes more beam-like and increases the complexity of room reflections.

Whatever strategy we adopt for analysing the acoustic radiation spectrum, we still can't easily state that one spectrum type is beautiful and can be moulded expressively by the player or that another is just OK and nothing special. However, if there is a clear defect that can be heard and is visible in the radiation spectrum, then we can relate this to the vibrating modes and have a basis for intervention. Most practically, these measuring techniques give us ways to accurately track changes and evaluate the results of any modifications.

The Low Frequency Region

Above about 600Hz there is a dip in the frequency response for all violins. It may extend up to 700 or 800Hz and vary in depth. Below this the radiation is dominated by 3 peaks: A0, $B1^-$ and $B1^+$. Peaks will sometimes be clear for CBR and A1 as well. These are the modes that a maker should become familiar with first. Below is a plot of the radiation of four high quality violins between about 240 and 650 Hz.



***Signature modes**: Acoustic Frequency Response of four fine violins, with similar features. (The levels are only approximate as no calibration was used) These modes exist for all violins though it is by no means always easy to identify them from the* *peaks is the spectrum. {See J Curtin, "Measuring Violin Sound", Strad3D and note (2) below}*

1. A0 (typically around c# on the g string) The lowest air mode and only strong radiator in violin's lowest octave. Essentially a Helmholtz resonance but modified by the compliance of the cavity walls. Radiation comes from the air motion in the *f*-holes. The cavity wall motion must have some volume change to excite this mode. Air flows from the *f*-holes at the same time as the cavity is contracting (and vice versa) subtracting from the radiation. However, the cavity wall motion needed to excite the mode is very small, and therefore there is only minimal cancellation.
2. CBR (**C**-**B**out-**R**homboid) (typically about an octave above the open g string) Lowest strong body mode. It shows shear-like, in-plane motions between top and back plates. Corpus motion is symmetric, having approximately equal areas in opposite phase. Air flow is in the opposite direction for each *f*-hole and in anti-phase to the wood surrounding each *f*-hole. It is therefore an inefficient radiator and normally causes only a small peak in the spectrum. *{see note (5)}*
3. $B1^-$ (typically around the open a string) Lowest strongly radiating corpus mode. Radiation strength depends on net volume change of the corpus. Areas of the surface that are adding to or subtracting from the volume are not equal. Additionally, air flows from the *f*-holes as the corpus expands reinforcing the radiation. (Bissinger suggests nominally 50% of acoustic radiation).
4. A1 (typically just above the open a string) Next higher air mode. A nodal line runs approximately across the length of the corpus in the middle of the *f*-holes. Volume flow is weak and tends to cancel within each *f*-hole. Therefore there is no significant radiation via the *f*-holes. Occasionally a strong radiator but only via induced wall motions (eg *Plowden* del Gesu). Frequency does not vary much between different violins and could be below a high-lying B1- or above a low-lying instance.
5. $B1^+$ (typically c – c# on the a string) Next strongly radiating corpus mode and usually stronger than $B1^-$. Radiation similarly dependent on volume change as for $B1^-$, but a different bending mechanism. When there is a "wolf note" it is it usually associated with this mode.

{For more detailed information see: Stoppani, "Shapes of the Signature Modes" and the Bissinger signature mode animations that include f-hole volume flows, Strad3D}.

Violins radiate sound because the wood moves when the strings vibrate and cause the air to move

- The vibration of the strings is transmitted almost entirely through the bridge, which acts as a filter. This means that all frequencies in the string vibrations are not transmitted to the body at the same level.
- We might add that without the soundpost some of the low frequency modes would look rather different and radiate much less effectively. The soundpost creates an asymmetry which modifies the modes in a way that leads to volume change during each cycle of vibration.

- Air modes are also excited by the wood motion. In the case of A0 (see above) the wood motion is very small in relation to the radiation from the air resonance. Higher air modes may or may not lead to radiation but they may well modify wood modes.
- The substructures (neck and fingerboard assembly and tailpiece) also have modes. Their modes may appear to be incorporated into corpus modes or to be independent. The lowest tailpiece modes sometimes seem to have no significant effect at all. Often neck-fingerboard and tailpiece activity are linked and the same activity in each may occur in different combinations with the other. Tuning these modes can have a significant effect on corpus modes. For example one tuning may greatly improve a “wolf note” and another make it much worse. Neck and fingerboard assembly and tailpiece modes are not confined to low frequencies but continue through the measurable range.
- The mechanisms for radiation are different for different frequencies. At A0 the radiation is dominated by the air activity. For $B1^-$ and $B1^+$ (and to a lesser extent for some higher modes) both plates and air are involved. As the mode frequencies get higher, it is increasingly only surface motion that causes radiation.
- Modes radiate with varying efficiency. *{See note (5)}* As for CBR (see above) when the areas in phase and anti-phase are similar cancellation takes place. This can also happen at higher up in the spectrum.

The level of sound radiation at different frequencies can be seen to relate to the level of vibration in the wood at those frequencies

The violin is a linear system or very nearly so. This means that if the input force is doubled the output is doubled. Whatever the mechanism for radiation, the wood activity increases in proportion to the input force, and if that excites an air mode its level will also increase in proportion. However, as stated above, modes do not all have the same radiating efficiency and the level of vibration in the wood at any particular frequency does not directly indicate the level of radiated sound. *{See note (5)}*

Even though the violin is a linear system this does not mean that a player will actually be able to put energy into the violin in proportion to the bowing effort. This is a playability issue.

The level of vibration in the wood is a direct consequence of the modes that exist in that violin (as excited by string vibrations transmitted through the bridge).

Modes need four numbers to describe them fully:

- A magnitude or amplitude at each measured point over the structure
- A phase angle indicative of the time relation between application of the driving force and the violin response, relative to the mode period (we generally expect plots and animations to show phase-antiphase motions).
- The resonance frequency
- The damping factor (a measure of how much of the energy is lost in each vibration cycle).

Some modes can be considered strong while others are weak enough to disregard for practical purposes. The maximum excitation occurs when the excitation frequency matches the resonance frequency of the mode, but a mode can be excited to a lesser degree by a frequency that is close to resonance and resonates at the excitation frequency (not the mode peak frequency). How close depends on the damping factor, which spreads the peak while it lowers the maximum response. The fundamental or harmonic of a string will more often fall either side of a resonance peak than very close to the peak frequency.

Modes are independent of each other and exist simultaneously. At any given frequency we just add up the contribution of all modes. Sometimes the response at a particular frequency will be overwhelmingly dominated by a single mode and at another consist of equal amounts of two modes. The contribution of a mode that is very far away in frequency from the excitation frequency will have a negligible effect but the summed effect of a large number of distant modes can be significant.

These modes are themselves a consequence of the chosen shape, construction details and the mass, stiffness and damping properties of the wood

The frequencies of modes result from the stiffnesses and masses of the component parts of the structure but not as directly as would be convenient. A rib garland without plates is very floppy. Free plates have free edges and the mode shapes as seen in the glitter patterns can no longer exist once the box is assembled. When glued together the box is much stiffer than its component parts and the reasons for the changes in stiffness are not immediately obvious. The arching of the plates can accommodate "out of plane" pumping motion, "in-plane" inward or outward motion at the edges and even more complex combinations of motion. As can be seen from the Polytec animations, the mode shapes include 3 dimensional motion and compression waves not apparent from a simpler form of modal analysis.

All the energy put into the violin, at a moment in time, by the string vibrations is dissipated in about half a second. Some of the energy is lost as sound radiated into the far field (radiation damping). The higher the proportion of input energy that goes this way the better: it means that violin is working as an effective sound-radiating machine!

The rest of the energy is lost in the wood, joints, the surrounding air and contact with the player or support fixtures for a measurement procedure. Losses due to the air in the near field, particularly air-flow through the *f*-holes, may involve turbulence. However, the greater part of the losses is in the wood. If we considered air to be the same for all violins then losses due to the air are also a consequence of the wood motion. The losses in the wood result from the deformations that occur for any particular mode. Any mode is likely to involve out-of-plane bending (along and across the grain and twist) as well as in-plane motion. Each mode has a unique shape and therefore a unique pattern of deformation and damping factor. There is quite a lot of scatter for mode damping factors, but it appears that damping gets, on average, lower with rising frequency.

As stated previously, the assumption that **"**These modes are themselves a consequence of the chosen shape, construction details and the mass, stiffness and damping properties of the wood" is the basis for finite element models of the violin. For a number of reasons there are practical limits as to how accurately a violin can be simulated. Wood properties are not uniformly distributed throughout a piece, even as small as a violin plate. Grain orientation would need to be included, as would the smaller effects of the air. In industry it is common practice to have a finite element model that is compared to experimental modal analysis (measurement of a real object). The differences between the two models are noted and the FEA model is refined or the measurement procedure revised. When the agreement is good enough there is a good basis on which to make modifications to the design.

It follows, in principal, that a desired set of modes could be achieved by appropriate selection of design and wood. While possible for simple structures, a violin is so complex that only very limited predictions would be possible. A much more likely scenario is that we make the violin according to our best guesses in the light of previous experience. The result will be essentially accidental but not entirely random because we can make some elementary predictions about the modes and the kind of tone that will result.

3. Some elementary predictions for tone with high or low-lying signature modes

The frequency of A0 may be a little higher or lower than average but this seems much less important than how strongly it radiates. CBR is often at a higher frequency in del Gesu models than for Strads. Since it is not a strong radiator we are not much concerned about its frequency except that if it is exceptional this may indicate a problem with the stiffness. The A1 frequency is dependent mainly on the length of the cavity so cannot be changed by more than a few Hz. It does not seem to be a prerequisite for good violins to have significant radiation from either CBR or A1 and not a problem if it does.

It has been suggested that the frequency of the $B1^+$ mode is an indicator of the tonal and playing character of violins. *{See note (6)}* If it is high-lying the violin is more soloistic but more resistant and at worst stubborn. If it is low-lying we would say it is compliant, warm and dark but maybe lacking vitality. It has also been observed that violins with high-lying $B1^+$ modes often have a low A0 response. The frequency spacing of the $B1^-$ and $B1^+$ modes is usually around 100 Hz: if one is high the other is likely to be high also. In this respect they are a pair of modes that tell us something about the dynamic stiffness of the violin. If the $B1^-$ - $B1^+$ delta is very small or very large there may be a gap in the radiation spectrum. This may be tracked to unusual cross-grain to long-grain stiffness ratio in either plate or to poorly matched arching. Bending plots show clearly that for $B1^-$ long-grain bending of the back and cross-grain bending of the top dominate, and the other way round for $B1^+$.

While we associate high-lying low-frequency modes with high arching and thicker plates, the wood properties are also a large factor. There is a positive correlation between wood density and stiffness but this does not mean that the ratio is always the same. Nor are the cross-grain, long-grain and twist stiffnesses always in the same proportion. These observations about the B1 frequencies are generalisations and we need to ask why these modes are high or low. If they are high because the wood has very low density the violin will probably not suffer from stubbornness or a weak A0. The excitation of A0 is dependent on the bridge rocking motion meeting sufficient compliance to allow volume change of the cavity. The dynamic stiffness could be still be moderate even with a high $B1^+$ mode. If the wood is thick and heavy (even with a low $B1^+$ mode) A0 will probably be weak and it will also sound scratchy and nasal.

The frequencies of the B1 modes are the easiest aspect of a violin's modes to adjust; simply reducing the thickness of either or both plates is likely to lower the frequencies of these two modes. Just as tuning a free plate is a way to compensate for variation in wood properties setting the B1s to within an appropriate range can be an effective adjustment for tone quality and the feel of the instrument. Of course, little is ever quite so simple in violin making. To say "high" or "low" lying implies that there is a "normal" or "ideal" frequency. "Normal", in this context would mean within the typical range for a type of violin and its characteristic tonal flavour. A high arched Amati model can be expected to have high lying B1s whereas a larger, low arched model would have them lower. It is possible to have one violin with exceptionally high B1s and another with them exceptionally low, yet they sound astonishingly similar. However, violins that have their B1 mode frequencies significantly outside the "normal" for their particular type are unlikely to be great instruments. If so it is likely that there is a problem with the arching, the wood quality or the graduations.

It is generally accepted that more success is to be had with wood that is at the low end of the density range. A piece of spruce may have low density but with very low stiffness or another somewhat higher density but exceptional stiffness. In this case the latter may be the better choice. In practice there is a range of density and stiffness that can result in successful violins but if either the density is too high or the stiffness too low we should expect a poor result. The worst result is likely to occur with very high density, low stiffness and thick plates. So a general strategy might be that, with low density wood that is not lacking in stiffness, a wider range of B1 mode frequencies is safe. If it seems stubborn then lowering the frequencies will probably help. If the wood is a little dense the violin will probably be more acceptable with thinner plates and lower lying frequencies. The reduction in plate mass would improve the response levels but the cost might be that only a somewhat dark tone can be achieved. If we are expecting to adjust the graduations of the plates after we have tried the violin with strings it is obviously much easier to start a little thicker. If the plates are judged too thin it is possible to add wood in the form of patches or "tone bars".

4. Conclusions

We have provided some basic information about violin modes and how they lead to sound radiation. We have also stated that the modes of a violin are a point of intervention for modifying the tone and playability. This presupposes that we have a concept of the tone we are aiming for and that we have some idea of what the changes we plan to make will do to the tone. We must be able to relate perceived tone to measured acoustic radiation before we can plan how to change the modes. We must also know what to do to change one or modes and the limits of what is possible. We are seeking to use a scientific approach to control the chain of consequences but with an imperfect toolbox. We could say that the new information is just an extension of the existing knowledge and expertise of violinmakers and provides more clues as to what would be an effective strategy to get a desired result.

We are all familiar with the effect that occurs as we reduce the graduation of a free plate: the tap tone pitches get lower. In just the same way, the modes of the whole violin tend to go down as the plates are thinned. Exactly how much the frequencies drop depends both on how much wood is removed and from which area. In addition to moving the frequency of a mode changes in graduation may also alter its shape and damping factor. We may wish to do this deliberately since such changes could affect near field cancellation and change the excitation level.

Careful selection of which areas to work on can enable a maker to have more effect on one mode than another, just as for free plates. Knowledge of the mode shapes is a great help in selecting areas for modification. If we have the equipment to make the measurements we would have information specifically for that violin but, if not, an understanding of a typical shape may still be helpful. Low frequency modes are more obstinate about changing shape because they are strongly dependent on the arching of the plates. Wood varies a great deal in its properties. Just because reducing an area by a certain amount resulted in a 20Hz drop in frequency for a mode in one violin there is no guarantee that this will always be so. In one case we might find it very sensitive to small changes and in another impossible to reach the target frequency without going unacceptably thin.

Thus deliberate modifications to violin tone are by no means easy and not necessarily always possible. It seems likely that, as familiarity with violin modes spreads within the violin-making culture, many empirical strategies will evolve. FEA model of violins already replicate the major features and we can expect improvements and important insights in the future. There is a vast amount of literature on the subject of violin acoustics. Much of it is written in rather technical language with recourse to mathematics and is therefore difficult to read for those without a technical background. I think it is not so much that the concepts are difficult as that the precise meaning of technical words may not be grasped and that mathematical notation presents a serious obstacle. Information that comes in the form of graphics, particularly animations of mode shapes, crosses the technical barrier. ~

6. Appendix

Note (1) Mode definitions. A weight hung on a spring bounces at a particular frequency because the elasticity of the spring and the momentum of the weight ‘balance’ resulting in a prolonged repetitive motion. An air mode will occur in a room when the wavelength of a sound matches the distance between two opposite walls. A sound wave is reflected from the walls causing regions that switch between high and low pressure but remain in the same place. (Called a ‘standing wave’) Thus modes can be thought of both in terms of masses and springs and as waves travelling in some medium. Whichever way you look at it modes store energy and require less energy to maintain a particular level of activity at their peak frequency than away from it. They all leak energy and fade away unless the energy is topped up.

Note (2) Acoustic radiation measurements. These acoustic frequency response measurements were made by tapping the corner of the bridge with a tiny hammer. The radiation was picked up with a microphone at a distance of about 30cm from the violin. The hammer tap approximates to an impulse that contains all frequencies in equal amounts. The hammer has a force sensor embedded behind the tip so that the input force is known. Since these measurements are Frequency Response Functions ('FRFs': the ratio of output to input) it does not matter if the tap is not a perfect impulse or how strong the tap was. 12 microphone positions were used in a circle round the violin. An average of 4 or 5 taps was taken at each point and then all 12 averaged to reduce the effects of radiation directionality. There is still some influence from the room reflections. *{See Note (7)}*

Note (3) Playability. The quality attribute "playability" means what it feels like to play an instrument: is it easy or difficult, how quickly does it speak, is there a bad wolf-note, what can it be persuaded to do .. etc. The time it takes for a bowed note to reach a stable Helmholtz motion is related the damping factor of modes that are excited by the fundamental or harmonics of that note. A string that has both ends fixed rigidly will behave in a slightly different way from when one or both ends are in motion. The well-known (and extreme) example is the wolf-note. The simplest explanation is that a strong resonance interferes with the bowing process.

Note (4) **"**PERCEPTUAL EFFECTS OF VIOLIN ACOUSTICAL MODIFICATIONS", Claudia Fritz in association with B.C.J. Moore (Experimental Psychology), J. Woodhouse (Engineering) and I. Cross (Music).

Note (5) "Efficiency", wavelength and area of radiating surface.
Efficiency, in a general mechanical sense, means how much of the energy put into a system, in one form, emerges at the other end in a different form. We put fuel in the tank of our car and see how far we travel. When used with respect to acoustic radiation it is not so much a matter of how much energy went into the system as how much radiation results from the movement of the parts of the object under consideration. The most often adopted approach is to take an area of the object, all of which moves in the same direction at the same time, and model it as an equivalent circular, baffled piston. (Imagine a loudspeaker set in a wall that extends to infinity in all directions.) For this model (pumping at a particular frequency) the radiation will be strong and highly directional if the diameter of the piston is much greater than the wavelength of sound in air. If the diameter of the piston is half the wavelength in air, or less, the radiation is more or less uniform in a hemisphere. The radiation power falls off by a factor of 4 (6dB) per octave as the frequency goes below where the piston diameter equals the half-wavelength in air.

If we have two identical speakers set next to each other in the wall moving equally, but in the opposite direction, we have dipole radiation. If the diameter of these two equivalent pistons is small compared to the wavelength in air of the frequency then the air will tend to slosh between one and the other without radiating into the far field. This is called "near field cancellation". All violin modes have nodal lines running across the surface of the plates and areas either side see-sawing in opposite directions.

Below is a table of sound wavelengths for some example frequencies, taking the velocity of sound to be 344 meters per second. (Wavelength = velocity/frequency) We can see that violins should not be effective sound radiating machines in the region of the open strings because the mode shapes cannot provide large enough equivalent pistons and there will inevitably be near field cancellation.

10 Hz : 34.4 meters
100 Hz : 3.44 meters
1000 Hz : 0.344 meters (34 cm)
10,000 Hz : 0.0344 meters (3.4 cm)
20,000 Hz : 0.0172 meters (1.72 cm)

Or in relation to fundamental pitch of the open strings of a violin:
g (196 Hz) : 1.76 meters
d (294 Hz) : 1.17 meters
a (440 Hz) : 0.78 meters
e (659 Hz) : 0.52 meters

Here are the same tables but with half the wavelengths:

10 Hz : 17.2 meters
100 Hz : 1.72 meters
1000 Hz : 0.172meters (17.2 cm)
10,000 Hz : 0.0172 meters (1.72 cm)
20,000 Hz : 0.0086 meters (8.6 mm)

Or in relation to the open strings of a violin:
g (196 Hz) : 0.88 meters
d (294 Hz) : 0.585 meters
a (440 Hz) : 0.39 meters
e (659 Hz) : 0.26 meters

The shapes of the B1 modes circumvent this problem by virtue of their volume change and the secondary effect of air-flow from the *f*-holes. The size of an object that approximates to a pulsating sphere is irrelevant: it is only the volume change that counts.

As we move up in frequency the modes tend to have more nodal lines, to have smaller areas in the same phase and to have less volume change or air-flow from the *f*-holes. *{See the 'band average movies'}* It becomes important that adjacent regions on the plates in opposite phase do not have equal areas or there will be significant near field cancellation. But as we move up in frequency the half-wavelength of sound in air gets shorter while the velocity of bending waves in the wood increases at a faster rate. At approximately 4 kHz the velocity of a bending wave along the grain of the spruce catches up with the velocity of sound in air. This is termed the "critical" or "coincidence" frequency. As we approach this condition the radiation efficiency increases in proportion to the square of the frequency and plateaus above it. Thus, well above the critical frequency mode shapes become irrelevant and only amplitude matters. It is clearly beneficial to have the critical frequency low rather than high as the rise in efficiency will be steeper and there will be greater benefit below this condition. The frequency at which this occurs is a matter of wood properties and graduation and is not affected at all by the plate archings

Note (6) In "Mode tuning for the violin maker" by Carleen M. Hutchins and Duane Voskuil CAS Journal vol. 2, No. 4 (Series II), Nov. 1993, pp 5 – 9, the A1/B1 delta is discussed. This is confusing because the radiation peak due to $B1^-$ is attributed to A1, the existence of $B1^-$ is not acknowledged and $B1^+$ is termed just B1. If we take a large delta to mean a high lying $B1^+$ mode then violins where it is high-lying are said to be suitable for soloists. Violins with lower $B1^+$ modes are for orchestral players, lower still for chamber players and very low easy to play but lacking in power.

Martin Schleske in "Empirical Tools in Contemporary Violin Making: Part I. Analysis of Design, Materials, Varnish, and Normal Modes" (CASJ Vol.4, No. 5 (Series II), May 2002) describes the $B1^+$ mode as a "tonal barometer":
"A B1 frequency below 510 Hz is characteristic of a somewhat "soft" violin with dark sound, lacking "resistance." In contrast, a B1 frequency above 550 Hz is found in "stubborn" violins with bright sound, possibly with a tendency to harshness, and with strong "resistance" to the player."

Note (7) When doing modal analysis of a violin, or measuring acoustic radiation, the most common practice is to consider the driving point to be the top of the bridge in a direction that corresponds to the lateral displacement of the strings when bowed. Of course, each string is bowed at a different angle and its motion is not guaranteed to be only parallel to the bow hair. It would be possible to take the measurements with a different driving point, or with several, but if resources permit only one driving point then this is a good choice. This can by justified because excitation via the bridge in the vertical direction is at a very much lower level than in the lateral direction and there do not appear to any modes that are only excited by vertical motion. To put it another way we are only interested in modes that can be excited when the strings vibrate and transmit their vibrations through the bridge to the body of the violin. In this way account is taken of the filtering properties of the bridge and the analysis will be a plausible representation of how much a particular mode can be excited by the string vibrations. The strings themselves have much less damping than the violin body. If measurements are taken with the strings allowed to vibrate freely it is all but impossible to disentangle the effect of the strings from the rest of the violin. Most researchers choose to damp the strings with business cards or bits of foam, though never 100% effective.